CHEM 100
Principles Of Chemistry

Chapter 2 - Quantitative Science

2.1 Quantitative Science

- Much of the power of science comes from its **quantitative** aspects
- From observations that Coke sinks in water and Diet Coke floats in water we can make two hypotheses

“Diet Coke is less dense than water”

“Coke is more dense than water”

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \qquad d = \frac{m}{V}$$

- Before we measure the density we must consider **units** and **measurements**

2.2 Scientific Units of Measure

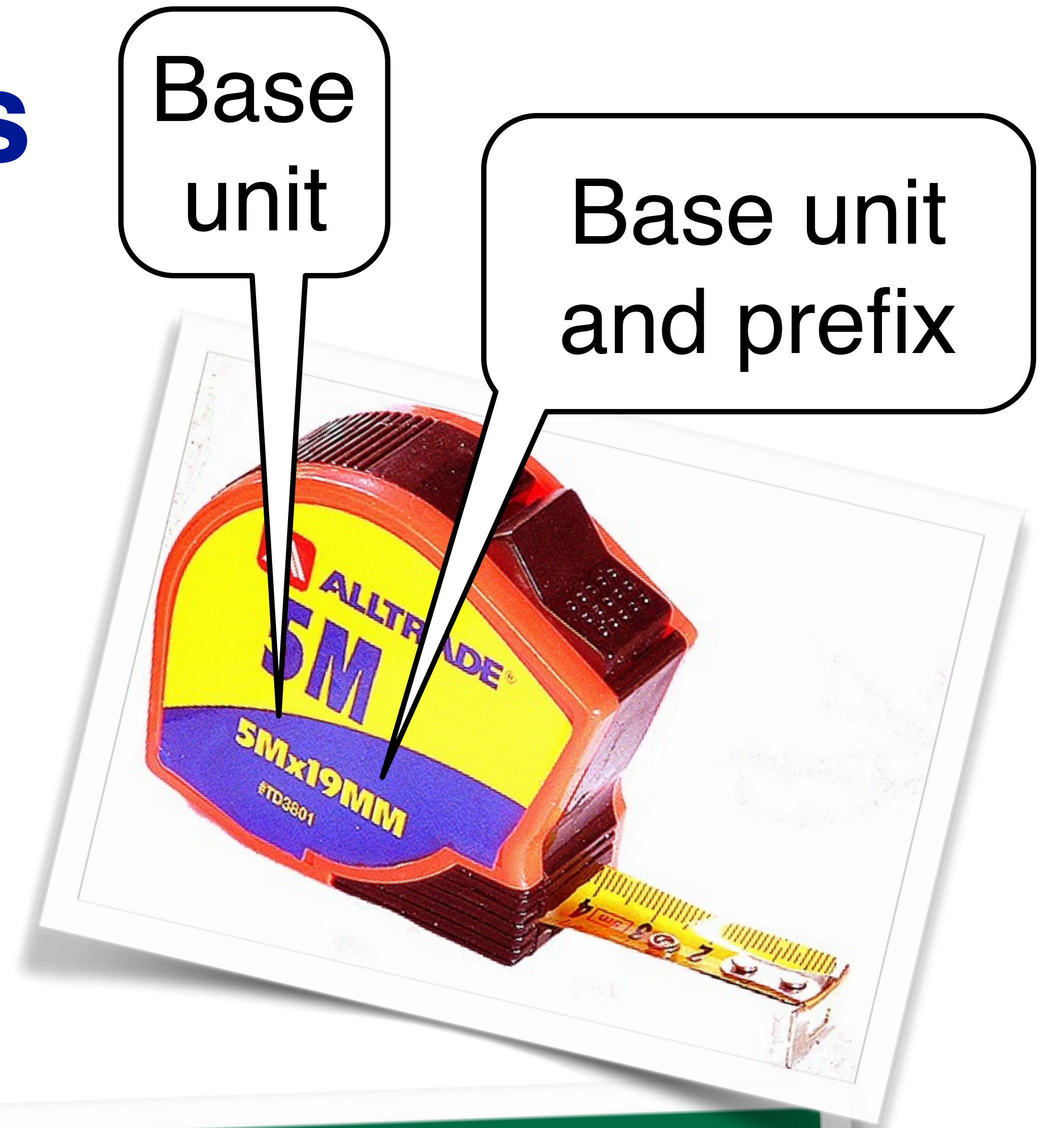
- In 1983 an Air Canada jet ran out of fuel at 26,000 feet between Montreal and Edmonton
  - It made an unpowered emergency landing at Gimli, Manitoba
- Crews loaded 22,300 pounds not 22,300 kg of fuel
  - The flight had less than half of the required fuel
- Specifying the **units** for a

The ‘Gimli Glider’

The plane was the first delivered with **metric** not **imperial** (English) unit fuel gauges

SI Units

- In science, units have been standardized into the **International System of Units** (**SI units**)
- Each quantity has a **base unit** and a **prefix** that multiplies the base quantity by a power of ten
  - Base units are quantities like meter, grams and liters
  - Prefixes are terms like milli, centi and kilo



Name	Symbol	Scaling Factor	Decimal Equivalent
Tera	T	x 10^{12}	1,000,000,000,000
Giga	G	x 10^{9}	1,000,000,000
Mega	M	x 10^{6}	1,000,000
Kilo	k	x 10^{3}	1,000
Base unit			
Centi	c	x 10^{-2}	0.01
Milli	m	x 10^{-3}	0.001
Micro	μ	x 10^{-6}	0.000001
Nano	n	x 10^{-9}	0.000000001
Pico	p	x 10^{-12}	0.000000000001

- Memorize these prefixes!

Unit Conversions

- We use **scaled units** (prefix plus base unit) because they are more convenient
  - 0.00056 km = 56 cm
  - 128,000,000 bytes = 128 Mbytes
- We often must **convert between units**
- To convert a <u>scaled unit to a base unit</u> simply replace the symbol with the scaling factor

Q What is 3.4 km in m?

A Replace 'k' with 'x 10^3'

3.4 <u>k</u>m = 3.4 <u>x 10^3</u> m (or 3,400 m)

Unit Conversions

- It is more difficult to turn a base unit into a scaled unit so we should generalize the procedure first
- To convert between any pair of units multiply by a **conversion factor**
- The conversion factor is a ratio chosen so that the numerator and denominator are the same quantity expressed in different units

$$3{,}400\ \cancel{\text{m}} \times \underbrace{\frac{1\,\text{km}}{10^3\ \cancel{\text{m}}}}_{\text{conversion factor}} = 3.4\ \text{km}$$

Always multiply by conversion factor

Same quantity on top and bottom!

Unit Conversions

- We can write two correct forms of the conversion factor but only one will give the correct units

$$3{,}400\ \text{m} \times \underbrace{\frac{1\ \text{km}}{10^3\ \text{m}}}_{\text{conversion factor}} = 3.4\ \text{km} \quad ✓$$

$$3{,}400\ \text{m} \times \underbrace{\frac{10^3\ \text{m}}{1\ \text{km}}}_{\text{conversion factor}} = 3{,}400{,}000\ \text{m}^2/\text{km} \quad ✗$$

- Use the units of the conversion factor to decide which is correct form

Test Yourself: Unit Conversions

Q What is 60 g in kg?

A Our conversion is g to kg so the conversion factor must include both units

$$60\ \text{g} \times \frac{10^3\ \text{g}}{1\ \text{kg}} = 60{,}000\ \text{g}^2/\text{kg} \quad \times$$

$$60\ \text{g} \times \frac{1\ \text{kg}}{10^3\ \text{g}} = 0.06\ \text{kg} \quad \checkmark$$

- Try these:

Q What is 4.35 kg in g? (4350 g)

Q What is 16.2 µm in m? (1.62×10^{-5} m or 0.0000162 m)

Unit Conversions

- It is more challenging to convert between <u>two scaled units</u>
- The key is to do the conversion in two steps, converting to the base unit first then the desired unit

Q What is 1.2 mm in µm?

A We'll convert 1.2 mm → m then m → µm

$$\text{mm} \rightarrow \text{m} \quad 1.2\ \text{mm} \times \frac{10^{-3}\ \text{m}}{1\,\text{mm}} = 0.0012\ \text{m}$$

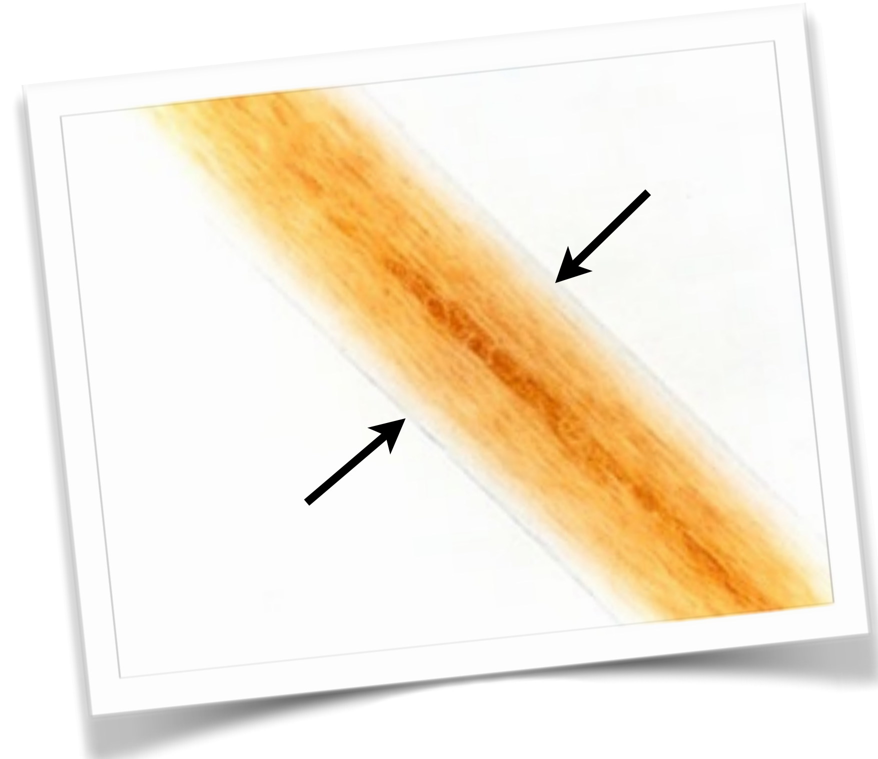
$$\text{m} \rightarrow \mu\text{m} \quad 0.0012\ \text{m} \times \frac{1\ \mu\text{m}}{10^{-6}\ \text{m}} = 1{,}200\ \mu\text{m}$$

Unit Conversions

- We can do the previous calculation in one step instead of two by multiplying the conversion factors
  - Either method is acceptable and should give the same answer!

Q What is 1.2 mm in µm?

A Converting 1.2 mm directly to µm gives



Human hair
40-250 µm

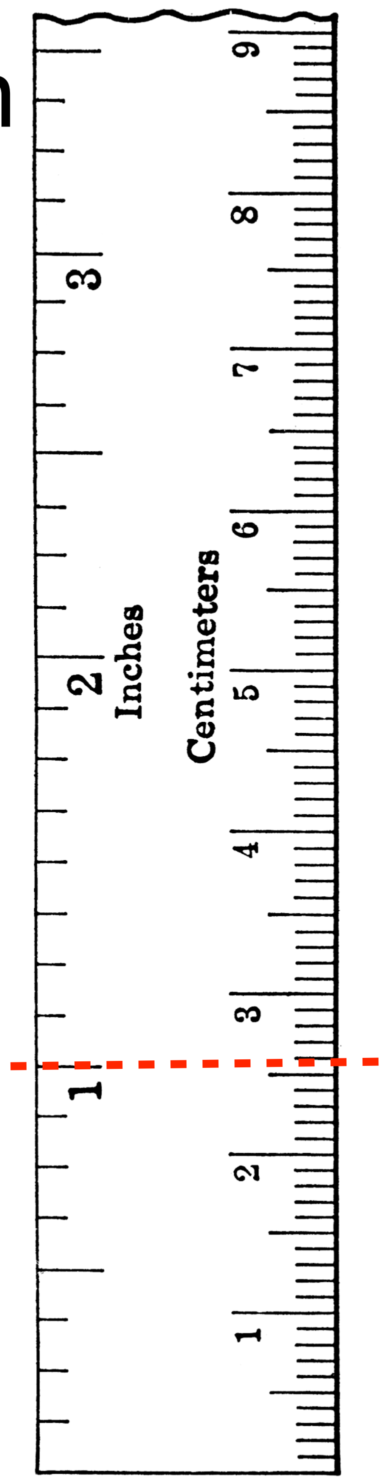
$$\text{mm} \rightarrow \mu\text{m} \quad 1.2\ \text{mm} \times \frac{10^{-3}\ \text{m}}{1\,\text{mm}} \times \frac{1\,\mu\text{m}}{10^{-6}\ \text{m}} = 1{,}200\ \mu\text{m}$$

Unit Conversions

- Sometimes it is necessary to convert between SI and non-SI units
- The same principles apply but a conversion factor must be given
  - These do not need to be memorized

Quantity	Conversion factor
Distance	1 inch = 2.54 cm
Mass	1 lb ≃ 454 g
Volume	1 gal ≃ 3.78 L

$$\frac{2.54\ \text{cm}}{1\ \text{inch}} \quad \text{or} \quad \frac{1\ \text{inch}}{2.54\ \text{cm}}$$



Test Yourself: Unit Conversions

Q Express 16.4 cm in feet given 1 inch = 2.54 cm and 1 foot = 12 inches

A We have two unit conversions, cm → inches → feet

A **solution map**

$$16.4\ \text{cm} \times \frac{1\ \text{inch}}{2.54\ \text{cm}} \times \frac{1\ \text{foot}}{12\ \text{inches}} = 0.538\ \text{feet}$$

Canceling units in solution map to give correct units of answer is **dimensional**

- Try these:

Q What is 10.5 gallons in L given 1 gal = 3.78 L? (39.7 L)

Q What is 2.2 lb in kg given 1 lb = 454 g? (1.0 kg)

Test Yourself: Unit Conversions

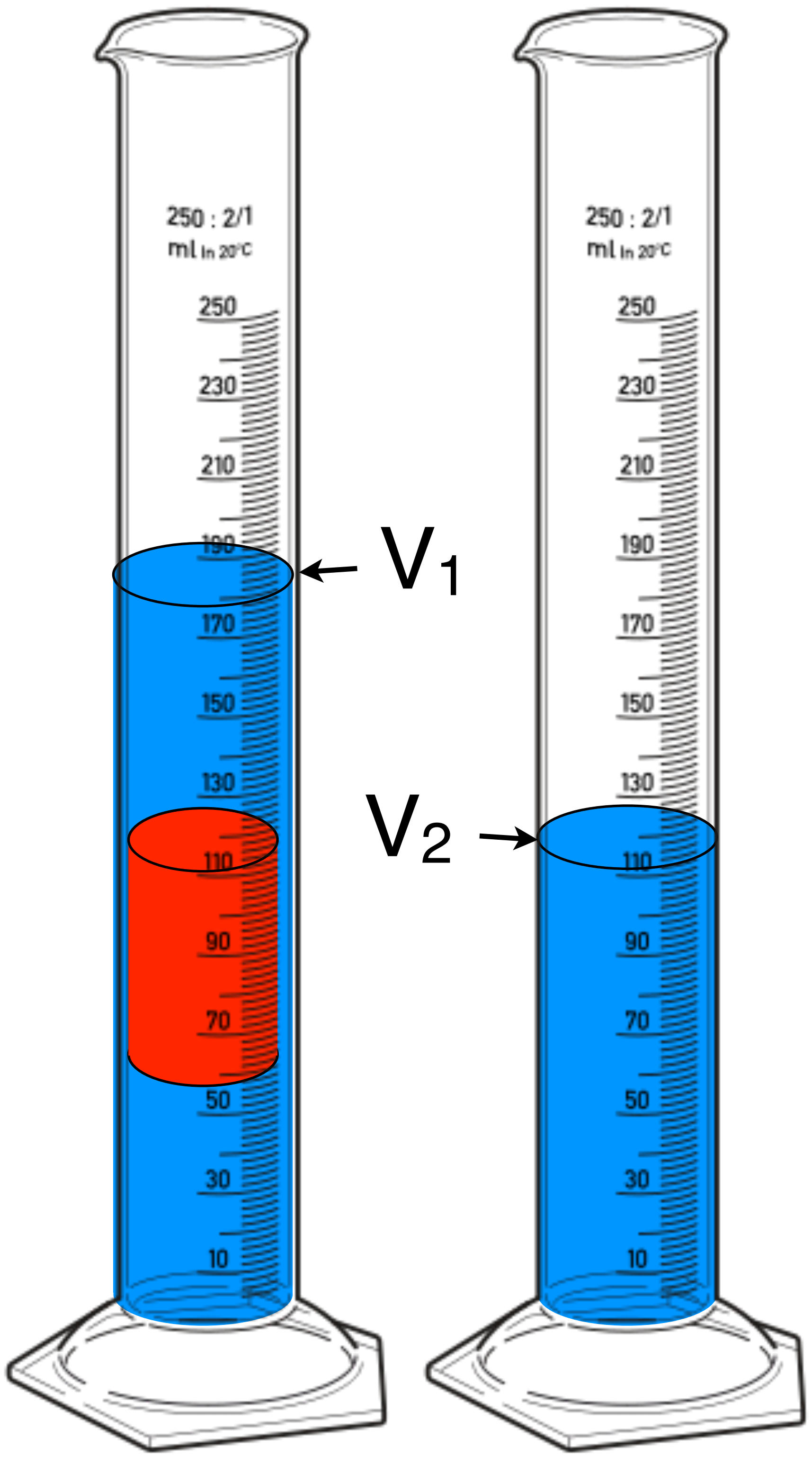
- Q Write the solution map for converting cm into mm (cm → m → mm)
- Q Convert 17 cm into mm (170 mm)
- Q Write the solution map for converting miles to km (miles → km)
- Q Convert 100 miles into km given that 1 km = 0.621 miles (161 km)
- Q Write the solution map for converting miles/hr to feet/s (mi/hr → ft/hr → ft/s)
- Q Convert 60 miles/hr into feet/s given that 1 foot = 1.89×10^{-4} miles) (88 feet/s)

The Soda Density Problem Revisited

- We can measure the mass m of the soda using a balance
- We can measure the volume V of the soda can by submerging it

$$V = V_1 - V_2$$

Sample	Mass (g)	Volume (mL)	Density (g/mL)
Coke	388.5	390	0.9962
Diet Coke	375.3	390	0.9623
Water (from reference table)			0.9978



The Soda Density Problem Revisited

"Diet Coke is less dense than water"

0.9623 g/mL 0.9978 g/mL

- This is verified so our hypothesis was correct ✓

"Coke is more dense than water"

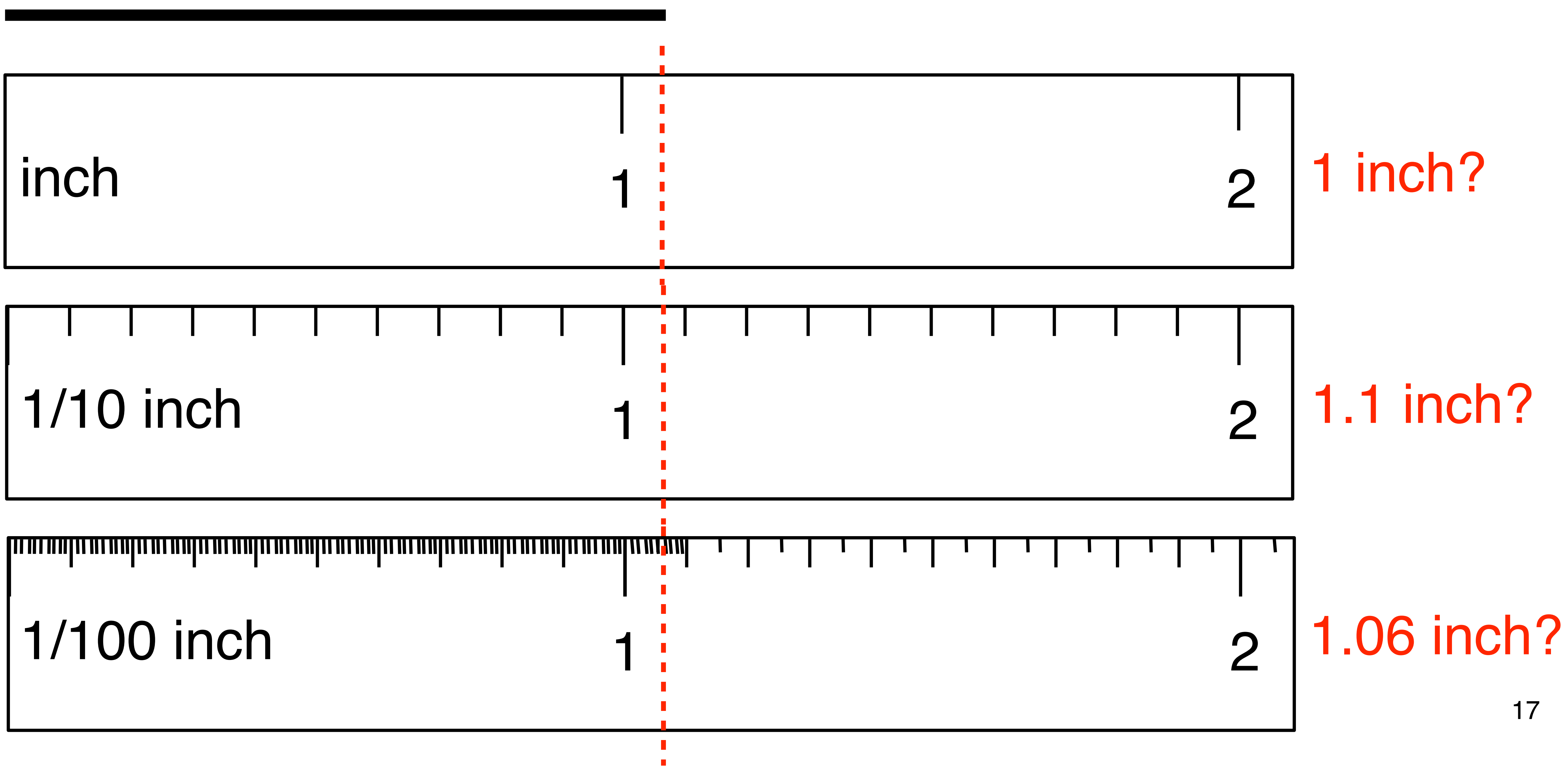
0.9962 g/mL 0.9978 g/mL

- This is not verified? Was our hypothesis incorrect? ✗

- The problem is that we were careless with our numbers

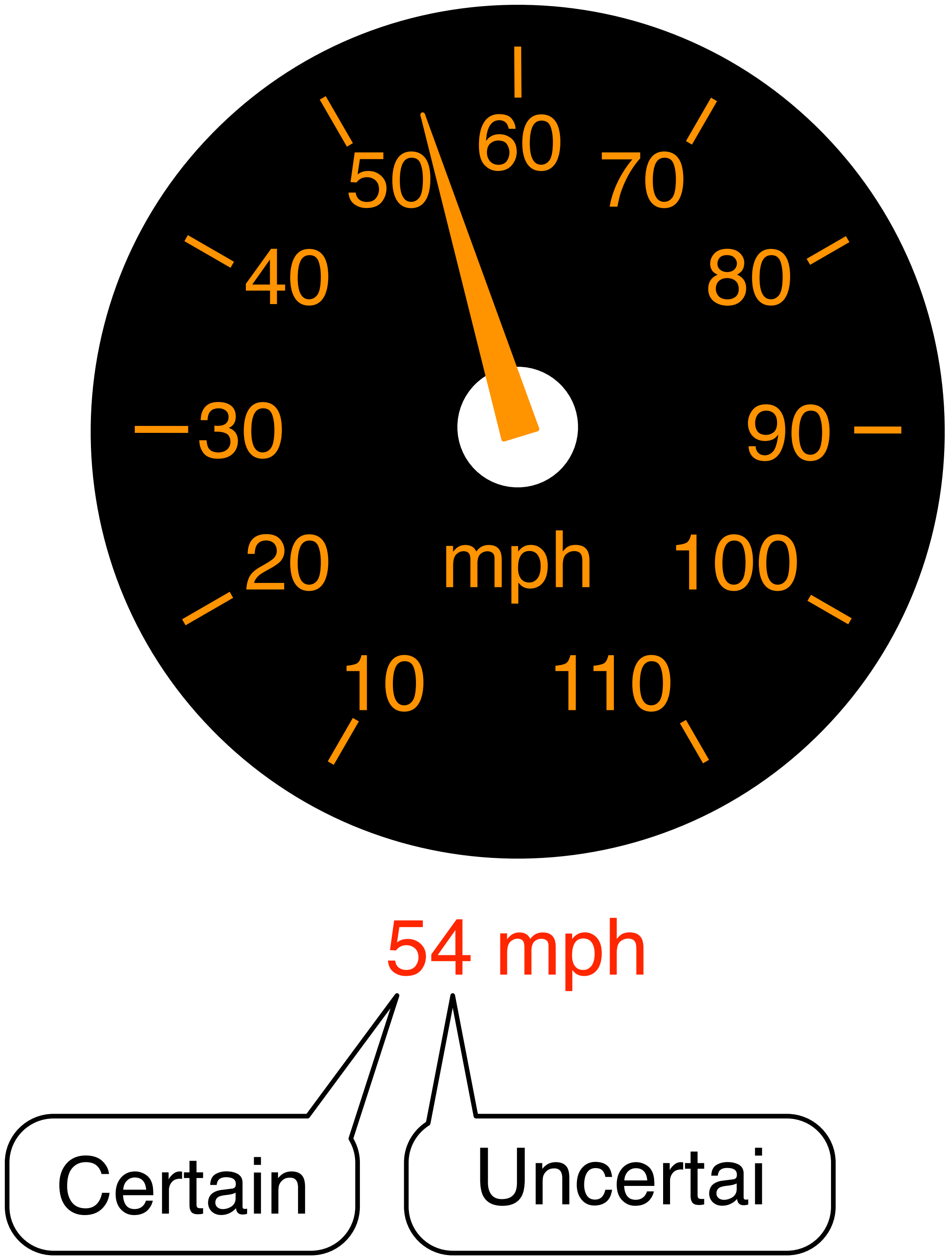
2.4 Significant Figures and Measurement Quality

- All measured quantities depend on the instrument with which they were measured



Significant Figures

- When scientists report measured quantities they give a measure of the quality by using **significant figures**
- Report **all certain digits plus one estimated digit**
  - We assume that the estimated digit is within ±1
- The estimated speed is 54 miles / hour
  - This implies the actual speed is in the range of 53-55 miles / hour



Significant Figures

- The more significant figures in a number, the more accurately the number is measured or known
- So how do we find the number of significant figures?
- **All digits are significant except zeros to the left of a number**

Number	Number of Significant Figures	Number	Number of Significant Figures
123	3	0.00123	3
1,234.5	5	0.01023	4
12,034	5	12,000	2 or 5?

Significant Figures

Q Does 12,000 have 2 or 5 significant figures

- Do I mean the number is between 11,000 and 13,000 or between 11,999 and 12,001? Or something in between?

A Report in **scientific notation**

- 1.2x10^4 has 2 significant figures but 1.2000x10^4 has 5 significant figures

A Specify the first uncertain digit with **underlining**

- $1\underline{2},000$ has 2 significant figures but $12,00\underline{0}$ has 5 significant figures

A Use a **decimal point**

- 12,000 has 2 significant figures but 12,000. has 5 significant

Test Yourself: How Many Significant Figures?

- How many significant figures in the following?

Number	Number of Significant Figures
5,600.02	6
7,230	3 or 4 ✗
7.20×10^6	3
0.010080	5
15,010	4
600.	3

Significant Figures: Exact Number

- Some numbers are **exact numbers**
  - 1 dozen = 12 (exactly)
  - 1 inch = 2.54 cm (exactly)
  - 1 mm = 10^{-3} m (exactly)
- Exact numbers have an infinite number of significant figures (or as many as needed)
- Exact numbers usually can be identified from the context of the question

1.000… dozen = 12.000...

2.5 Calculating With Significant Figures

- Many quantities are derived from calculations of measured quantities
- Surely an answer cannot be 'better quality' than the numbers from which it was calculated?
- We should <u>never</u> use all the numbers from a calculator
- Calculations must be **rounded off** to an appropriate number of significant figures

$$d = \frac{m}{V}$$

$$= \frac{388.5\ \text{g}}{390\ \text{mL}}$$

$$= 0.996153846\ \text{g / mL}$$

Rounding Numbers

- Identify the **rounding digit**, the last number that will appear in the rounded answer
- If the rounding digit is followed by 0, 1, 2, 3 or 4, simply cut off all following digits
  - Rounding 2.44 to 2 significant figures = 2.4
- If the rounding digit is followed by 6, 7, 8 or 9, add one to the rounding digit and cut off all following digits
  - Rounding 128.7 to 3 significant figures = 129
- If the rounding digit is followed by 5 (exactly) the rounding digit becomes the nearest even number
  - Rounding 2052.5 to 4 significant figures = 2052

Significant Figures in Multiplication and Division

- **Rule: For multiplication or division, round off the answer to the same number of significant figures in the measurement with the fewest significant figures**
- Give the answer to (1.23 x 2.4) ÷ 1.0030 with the correct number of significant figures

3 s.f.

2 s.f.

12 s.f.

$$\frac{1.23 \times 2.4}{1.0030} = 2.94317048853$$

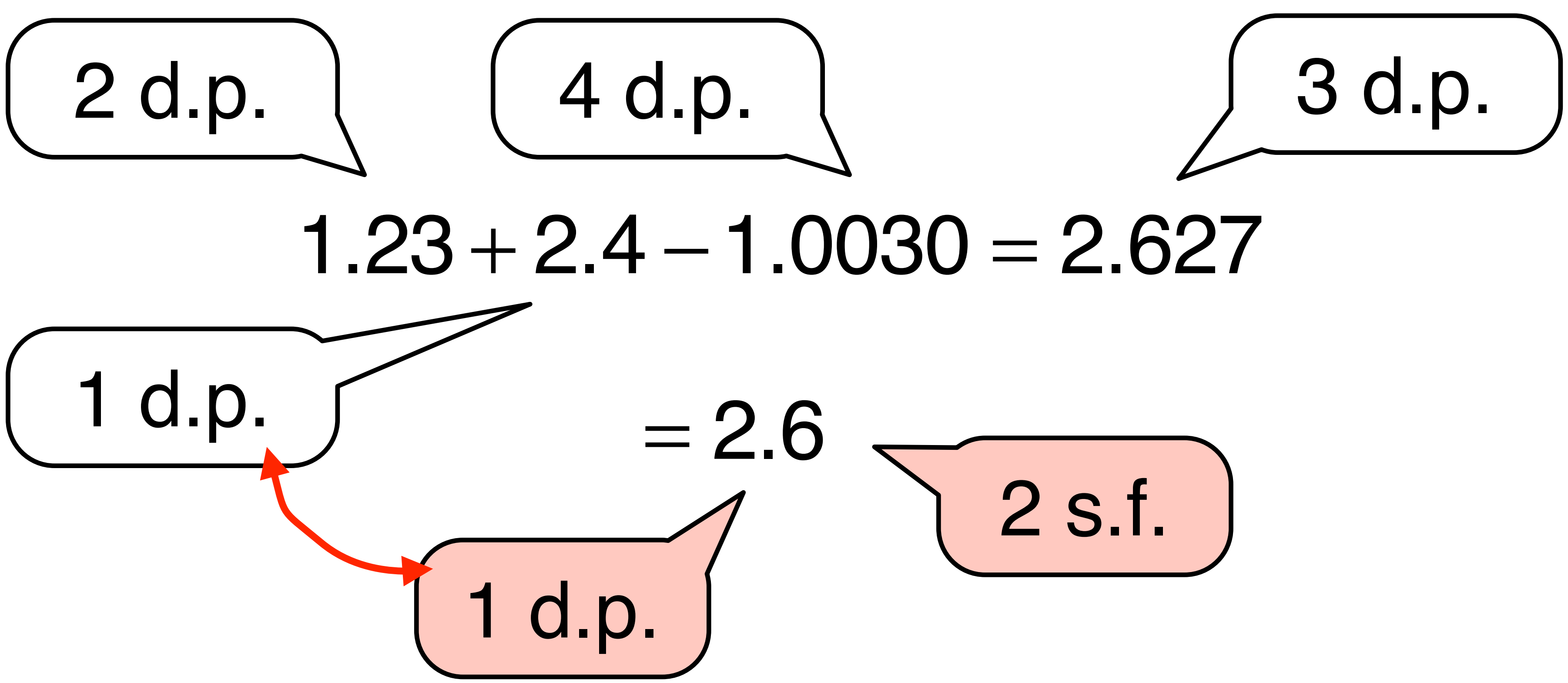
5 s.f.

$$= 2.9$$

2 s.f.

Significant Figures in Addition and Subtraction

- **Rule: For addition or subtraction, round off the answer to the same number of decimal places in the measurement with the fewest number of decimal places**
- Give the answer to (1.23 + 2.4) − 1.0030 with the correct number of significant figures



More Challenging Examples

Q Report 9.55 + 1 with the correct number of significant figures

A

0 d.p.

Round this answer to 0 d.p.

$$9.55 + 1 = 10.55 = 11$$

2 s.f.

- Note that the answer has 2 s.f. but one of the original numbers only has 1 s.f.!

More Challenging Examples

Q Report the average of 9.2, 9.45 and 9.003 with the correct number of significant figures

A

1 d.p.

This answer should have 1 d.p. (or 3 s.f. in this case) but don’t round yet!

3 s.f.

$$\frac{9.2+9.45+9.033}{3}=\frac{27.683}{3}=9.22766...=9.23$$

∞ s.f. because it is exact number!

Round this answer to 3 s.f.

The Soda Density Problem Revisited

- How many significant figures should we use to report density of the soda?

$$d = \frac{m}{V}$$

Sample	Mass (g)	Volume (mL)	Density (g/mL)	Rounded Density (g/mL)
Coke	388.5	3.9×10^2	0.9962	1
Diet Coke	375.3	3.9×10^2	0.9623	0.96
Water (from reference table)			0.9978	

"Diet Coke is less dense than water" ✓

"Coke is more dense than water" ✓

2.6 Problem-Solving Strategies for Numerical Problems

- Some questions are much more involved than the simple ones given so far

1. Read the entire problem
2. What is asked for and what units are required?
3. Note any quantities (numbers and units)
4. Think about relationships between quantities and create a solution map
5. What conversion factors are needed and given?
6. Calculate carefully and confirm that units cancel to give desired units (dimensional analysis)
7. Does the answer make sense?

Review: Learning Objectives

- Express measurements in scientific units (Section 2.2; Exercise 1)
- Create and use solution maps (Section 2.2; Exercise 7)
- Convert between common American units and scientific units (Section 2.2, 2.3; Exercises 6, 8-12)
- Determine the number of significant figures in a measurement (Section 2.4, 2.5; Exercise 2)
- Round to the appropriate number of significant figures (Section 2.5; Exercises 3-5, 8)
- Apply a systematic approach to solving problems (Section 2.6; Exercises 13-16)